King Fahd University of Petroleum and Minerals
College of Computer Sciences and Engineering
Information and Computer Science Department

ICS 253: Discrete Structures I
Summer semester 2016-2017
Major Exam #1, Saturday July 22, 2017
Time: **100** Minutes

Name: ____

ID#: ______________________

Instructions:
1. The exam consists of 8 pages, including this page, containing 7 questions.
2. Answer all 7 questions. Show all the steps.
3. Make sure your answers are **clear** and **readable**.
4. The exam is closed book and closed notes. No calculators or any helping aides are allowed. Make sure you turn off your mobile phone and keep it in your pocket.
5. If there is no space on the front of the page, use the back of the page.

Question	Maximum Points	Earned Points	Remarks
1	20		
2	15		
3	10		
4	10		
5	25		
6	10		
7	10		
Total	**100**		

Rules of Inference:

$p \to (p \vee q)$	Addition	$[\neg q \wedge (p \to q)] \to \neg p$	Modus Tollens
$(p \wedge q) \to p$	Simplification	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$[(p) \wedge (q)] \to (p \wedge q)$	Conjunction	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive syllogism
$[p \wedge (p \to q)] \to q$	Modus Ponens	$[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$	Resolution
$\forall x P(x) \to P(a)$ for all a	Universal Instantiation	$\exists x P(x) \to P(a)$ for some a	Existential Instantiation
$P(a)$ for all a $\to \forall x P(x)$	Universal Generalization	$P(a)$ for some a $\to \exists x P(x)$	Existential Generalization

Q1: [20 points] Answer the following questions.

a) [4 points] Write each of these statements in the form "if *p*, then *q*" in English.

i. It is necessary to walk 8 miles to get to the top of Long's Peak.

If you get to the top of Long's Peak, then you have walked 8 miles.

ii. You will reach the top, unless you begin your climb too late.

If you begin your climb early, then you will reach the top.

b) [4 points] Determine whether $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are logically equivalent or not. Justify your answer.

They are not equivalent, since if $p = F$, $q = T$ and $r = F$, then the LHS evaluates to True whereas the RHS evaluates to False.

c) [6 points] Show that $((\neg p \vee q) \wedge p) \rightarrow q$ is a tautology without using a truth table.

$$
\begin{aligned}
\big((\neg \boldsymbol{p} \vee \boldsymbol{q}) \wedge \boldsymbol{p}\big) \rightarrow \boldsymbol{q} &\Leftrightarrow \big((\neg \boldsymbol{p} \wedge \boldsymbol{p}) \vee (\boldsymbol{q} \wedge \boldsymbol{p})\big) \rightarrow \boldsymbol{q} \\
&\Leftrightarrow \big(\boldsymbol{F} \vee (\boldsymbol{p} \wedge \boldsymbol{q})\big) \rightarrow \boldsymbol{q} \\
&\Leftrightarrow (\boldsymbol{p} \wedge \boldsymbol{q}) \rightarrow \boldsymbol{q} \\
&\Leftrightarrow \neg(\boldsymbol{p} \wedge \boldsymbol{q}) \vee \boldsymbol{q} \\
&\Leftrightarrow (\neg \boldsymbol{p} \vee \neg \boldsymbol{q}) \vee \boldsymbol{q} \\
&\Leftrightarrow \neg \boldsymbol{p} \vee (\neg \boldsymbol{q} \vee \boldsymbol{q}) \\
&\Leftrightarrow \neg \boldsymbol{p} \vee \boldsymbol{T} \\
&\Leftrightarrow \boldsymbol{T}
\end{aligned}
$$

d) [6 points] Given a conditional statement $p \longrightarrow q$, find the inverse of its inverse, the inverse of its converse, and the inverse of its contrapositive.

Inverse of its inverse:

The inverse of the statement $\boldsymbol{p} \rightarrow \boldsymbol{q}$ is $\neg \boldsymbol{p} \rightarrow \neg \boldsymbol{q}$. Hence, the answer is

$$\boldsymbol{p} \rightarrow \boldsymbol{q}$$

Inverse of its converse:

The converse of the statement $\boldsymbol{p} \rightarrow \boldsymbol{q}$ is $\boldsymbol{q} \rightarrow \boldsymbol{p}$. Hence, the answer is

$$\neg \boldsymbol{q} \rightarrow \neg \boldsymbol{p}$$

Inverse of its contrapositive:

The contrapositive of the statement $\boldsymbol{p} \rightarrow \boldsymbol{q}$ is $\neg \boldsymbol{q} \rightarrow \neg \boldsymbol{p}$. Hence, the answer is

$$\boldsymbol{q} \rightarrow \boldsymbol{p}$$

Q2: [15 points] Consider the following statements: "If Nada does not take a course in discrete mathematics, then she will not graduate." "If Nada does not graduate, then she is not qualified for the job." "If Nada reads this book, then she is qualified for the job." "Nada does not take a course in discrete mathematics but she reads this book."

a) [9 points] Translate the above statements to propositional logic.

$p \equiv$ Nada takes a course in discrete mathematics
$q \equiv$ Nada will graduate
$r \equiv$ Nada is qualified for the job.
$s \equiv$ Nada reads the book.

1- $\neg p \rightarrow \neg q$
2- $\neg q \rightarrow \neg r$
3- $s \rightarrow r$
4- $\neg p \land s$

b) [6 points] Determine whether these statements are consistent or not. Clearly justify your answer using logic.

$\neg p \land s \rightarrow s$	**(Simplification)**
$(s \rightarrow r) \land s \rightarrow r$	**(Modus ponens)**
$\neg p \land s \rightarrow \neg p$	**(Simplification)**
$(\neg p \rightarrow \neg q) \land \neg p \rightarrow \neg q$	**(Modus ponens)**

However, $\neg q \rightarrow \neg r \Leftrightarrow T \rightarrow F \Leftrightarrow F$
Hence, the system is inconsistent.

Q3: [10 points] Given the following Prolog facts and rule:

```
father (khalid, ahmad).
father(khalid, ali).
father(khalid, salim).
father(ahmad, layla).
father(ahmad, huda).
father(riyadh, nadir).
mother (nadia, ahmad).
mother (nadia, ali).
mother (salma, salim).
mother (noora, layla).
mother (noora, huda).
mother(noora, khalil).
mother (sameera, nadir).
mother(noora, khalil).
sibling(A, B) :- father(C, A), father(C, B).
```

where `father(x,y)` means that `x` is the father of `y`, `mother(z, w)` means that `z` is the mother of `w`, and `sibling(A, B)` is a rule as defined above between `A` and `B`. What would Prolog return when given these queries?

a) [2 points] `father(sameera, nadir).`

No (or False)

b) [2 points] `mother(X, layla).`

X = noora

c) [3 points] `father(khalid, Z).`

Z=ahmad
Z=ali
Z=salim

d) [3 points] `sibling(layla, X).`

X=layla
X=huda

Q4: [10 points] On the island of knights and knaves, suppose that you meet three people, Ahsan, Badr, and Rami. What are Ahsan, Badr, and Rami if Ahsan says "I am a knave and Badr is a knight" and Badr says "Exactly one of the three of us is a knight"?

If Ahsan is a knight, then his statement "I am a knave and Badr is a knight" should evaluate to true, implying that Ahsan is a knave, which is inconsistent.

If Ahsan is a knave, then his statement "I am a knave and Badr is a knight" should evaluate to false, but since he is a knave, the statement Badr is a knight, has to be false for the statement to be false. Therefore, Badr is a knave.

Badr's statement "Exactly one of the three of us is a knight" should be false, implying that either none of them is a knight, or two or more are knights. Since two of them are already knaves, the only possible case left is him being a knave.

Therefore, all of them are knaves.

Q5: [25 points] Let $T(x)$ be the statement "x is a student in my class", $P(x, y)$ be the statement "x has posted Post y on facebook," and $S(x, z, y)$ be the statement "x has shared Post y that was posted by z," where the domain for the variables x and z consists of all people, and the domain of variable y is articles, images and posts that are posted on facebook. Use quantifiers and predicates to express each of these statements, where no negation is outside a quantifier or an expression involving logical connectives. You are **not** allowed to use "∃!".

a) [5 points] Ahmad has shared some of the articles posted by Salim.

$$\exists y\, S(Ahmad, Salim, y)$$

b) [5 points] Not all students in my class have posted something on facebook.

$$\neg\forall x\exists y\,(T(x) \rightarrow P(x,y)) \Leftrightarrow \exists x\forall y(T(x) \wedge \neg P(x,y))$$

c) [5 points] Khalid shares every facebook post that is posted by Ali.

$$\forall y\, S(Khalid, Ali, y)$$

d) [5 points] There is exactly one student in my class who has shared his own posts.

$$\exists x\left(\exists y\,(T(x) \wedge S(x,x,y)) \wedge \neg\exists w\left(\exists z(T(w) \wedge S(w,w,z) \wedge (w \neq x))\right)\right)$$

$$\Leftrightarrow \exists x\left(\exists y\,(T(x) \wedge S(x,x,y)) \wedge \forall w\left(\forall z(\neg T(w) \vee \neg S(w,w,z) \vee (w = x))\right)\right)$$

$$\Leftrightarrow \exists x\left(\exists y(T(x) \wedge S(x,x,y)) \wedge \forall w\left(\forall z\left((T(w) \wedge S(w,w,z)) \rightarrow (w = x)\right)\right)\right)$$

e) [5 points] Waheed, who is not a student in my class, shares all posts that are posted by students in my class.

$$\neg T(Waheed) \wedge \forall y\forall z(T(y) \rightarrow S(Waheed, y, z))$$

Q6: [10 points] Answer the following questions.

a) (2 points) Find a domain for the quantifiers in $\exists x \exists y((x \neq y) \land \forall z((z = x) \lor (z = y)))$ such that this statement is false.

***U* is any set that does not contain only 2 elements.**

b) (8 points) Suppose that the domain of the propositional function *P(x)* consists of −5, −3, −1, 1, 3, and 5. Express the statement $\exists x(\neg P(x)) \land \forall x((x < 0) \rightarrow P(x))$ without using quantifiers, instead using only negations, disjunctions, and conjunctions.

$$\left(\neg \boldsymbol{P}(-\mathbf{5}) \lor \neg \boldsymbol{P}(-\mathbf{3}) \lor \neg \boldsymbol{P}(-\mathbf{1}) \lor \neg \boldsymbol{P}(\mathbf{1}) \lor \neg \boldsymbol{P}(\mathbf{3}) \lor \neg \boldsymbol{P}(\mathbf{5})\right) \land \left(\boldsymbol{P}(-\mathbf{5}) \land \boldsymbol{P}(-\mathbf{3}) \land \boldsymbol{P}(-\mathbf{1})\right)$$

Q7: [10 points] Use rules of inference to show that if

$$\forall x(P(x) \lor Q(x)) \text{ and } \forall x((\neg P(x) \land Q(x)) \rightarrow R(x))$$

are true, then $\forall x(\neg R(x) \rightarrow P(x))$ is also true, where the domains of all quantifiers are the same.

$$\forall \boldsymbol{x}\left((\neg \boldsymbol{P}(\boldsymbol{x}) \land \boldsymbol{Q}(\boldsymbol{x})) \rightarrow \boldsymbol{R}(\boldsymbol{x})\right) \rightarrow (\neg \boldsymbol{P}(\boldsymbol{c}) \land \boldsymbol{Q}(\boldsymbol{c})) \rightarrow \boldsymbol{R}(\boldsymbol{c}) \qquad (\mathbf{1})$$

$$\Leftrightarrow \neg \boldsymbol{R}(\boldsymbol{c}) \rightarrow \neg(\neg \boldsymbol{P}(\boldsymbol{c}) \land \boldsymbol{Q}(\boldsymbol{c})) \qquad (\mathbf{2})$$

$$\Leftrightarrow \neg\neg \boldsymbol{R}(\boldsymbol{c}) \lor \boldsymbol{P}(\boldsymbol{c}) \lor \neg \boldsymbol{Q}(\boldsymbol{c}) \qquad (\mathbf{3})$$

$$\Leftrightarrow \boldsymbol{R}(\boldsymbol{c}) \lor \boldsymbol{P}(\boldsymbol{c}) \lor \neg \boldsymbol{Q}(\boldsymbol{c}) \qquad (\mathbf{4})$$

$$\forall \boldsymbol{x}(\boldsymbol{P}(\boldsymbol{x}) \lor \boldsymbol{Q}(\boldsymbol{x})) \rightarrow \boldsymbol{P}(\boldsymbol{c}) \lor \boldsymbol{Q}(\boldsymbol{c}) \qquad (\mathbf{5})$$

From (4) and (5) and using the resolution inference rule, we get

$$\boldsymbol{R}(\boldsymbol{c}) \lor \boldsymbol{P}(\boldsymbol{c}) \lor \boldsymbol{P}(\boldsymbol{c})$$

$$\Leftrightarrow \boldsymbol{R}(\boldsymbol{c}) \lor \boldsymbol{P}(\boldsymbol{c})$$

$$\Leftrightarrow \neg \boldsymbol{R}(\boldsymbol{c}) \rightarrow \boldsymbol{P}(\boldsymbol{c})$$

$$\Leftrightarrow \forall \boldsymbol{x}(\neg \boldsymbol{R}(\boldsymbol{x}) \rightarrow \boldsymbol{P}(\boldsymbol{x}))$$